OCR
RECOGNISING ACHIEVEMENT

Tuesday 18 June 2013 – Morning

A2 GCE MATHEMATICS (MEI)

4753/01 Methods for Advanced Mathematics (C3)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4753/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

4715660613

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.
- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.
- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (36 marks)

1 Fig. 1 shows the graphs of $y = |x|$ and $y = a|x + b|$, where a and b are constants. The intercepts of $y = a|x + b|$ with the x- and y-axes are $(-1, 0)$ and $\left(0, \frac{1}{2}\right)$ respectively.

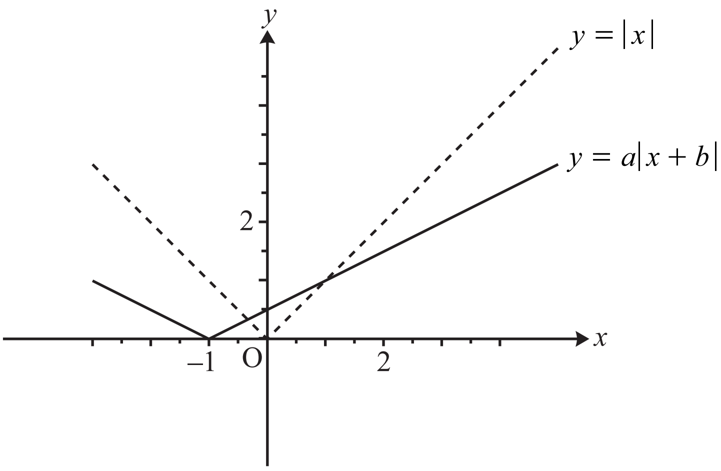

Fig. 1

(i) Find a and b. **[2]**

(ii) Find the coordinates of the two points of intersection of the graphs. **[4]**

2 **(i)** Factorise fully $n^3 - n$. **[2]**

(ii) Hence prove that, if n is an integer, $n^3 - n$ is divisible by 6. **[2]**

3 The function $\mathrm{f}(x)$ is defined by $\mathrm{f}(x) = 1 - 2\sin x$ for $-\frac{1}{2}\pi \leqslant x \leqslant \frac{1}{2}\pi$. Fig. 3 shows the curve $y = \mathrm{f}(x)$.

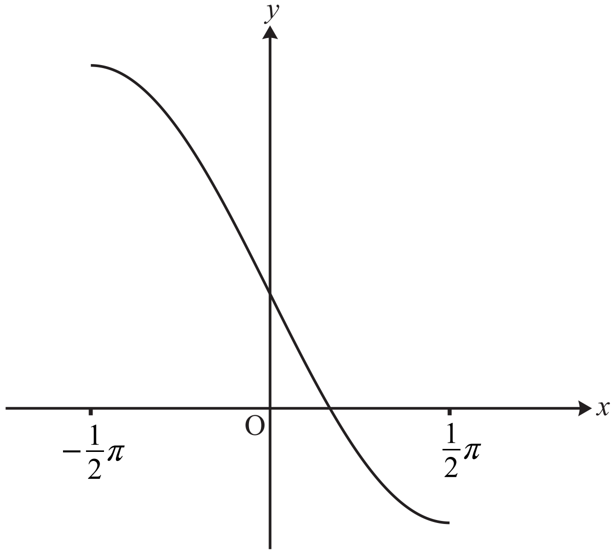

Fig. 3

(i) Write down the range of the function $\mathrm{f}(x)$. **[2]**

(ii) Find the inverse function $\mathrm{f}^{-1}(x)$. **[3]**

(iii) Find $\mathrm{f}'(0)$. Hence write down the gradient of $y = \mathrm{f}^{-1}(x)$ at the point $(1, 0)$. **[3]**

4 Water flows into a bowl at a constant rate of $10\,\mathrm{cm}^3\,\mathrm{s}^{-1}$ (see Fig. 4).

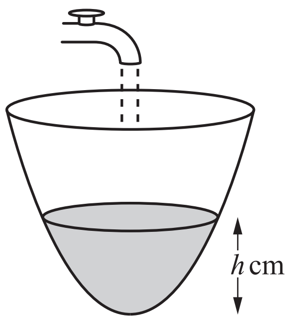

Fig. 4

When the depth of water in the bowl is h cm, the volume of water is $V\,\mathrm{cm}^3$, where $V = \pi h^2$. Find the rate at which the depth is increasing at the instant in time when the depth is 5 cm. **[5]**

5 Given that $y = \ln\left(\sqrt{\dfrac{2x-1}{2x+1}}\right)$, show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{2x-1} - \dfrac{1}{2x+1}$. **[4]**

6 Using a suitable substitution or otherwise, show that $\displaystyle\int_0^{\frac{1}{2}\pi} \frac{\sin 2x}{3 + \cos 2x}\,\mathrm{d}x = \frac{1}{2}\ln 2$. **[5]**

7 **(i)** Show algebraically that the function $\mathrm{f}(x) = \dfrac{2x}{1 - x^2}$ is odd. **[2]**

Fig. 7 shows the curve $y = \mathrm{f}(x)$ for $0 \leqslant x \leqslant 4$, together with the asymptote $x = 1$.

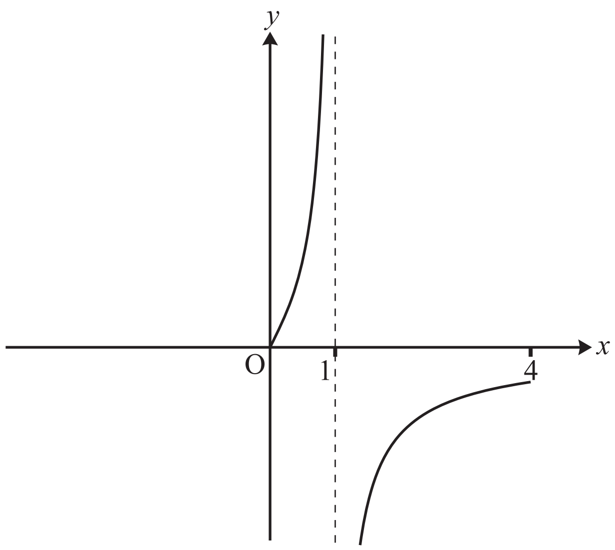

Fig. 7

(ii) Use the copy of Fig. 7 to complete the curve for $-4 \leqslant x \leqslant 4$. **[2]**

Section B (36 marks)

8 Fig. 8 shows the curve $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = (1 - x)\mathrm{e}^{2x}$, with its turning point P.

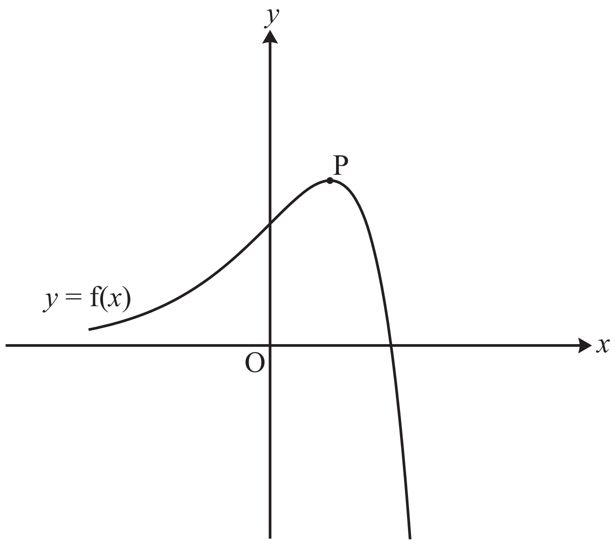

Fig. 8

(i) Write down the coordinates of the intercepts of $y = \mathrm{f}(x)$ with the x- and y-axes. **[2]**

(ii) Find the exact coordinates of the turning point P. **[6]**

(iii) Show that the exact area of the region enclosed by the curve and the x- and y-axes is $\frac{1}{4}(\mathrm{e}^2 - 3)$. **[5]**

The function $\mathrm{g}(x)$ is defined by $\mathrm{g}(x) = 3\mathrm{f}\left(\frac{1}{2}x\right)$.

(iv) Express $\mathrm{g}(x)$ in terms of x.

Sketch the curve $y = \mathrm{g}(x)$ on the copy of Fig. 8, indicating the coordinates of its intercepts with the x- and y-axes and of its turning point. **[4]**

(v) Write down the exact area of the region enclosed by the curve $y = \mathrm{g}(x)$ and the x- and y-axes. **[1]**

9 Fig. 9 shows the curve with equation $y^3 = \dfrac{x^3}{2x-1}$. It has an asymptote $x = a$ and turning point P.

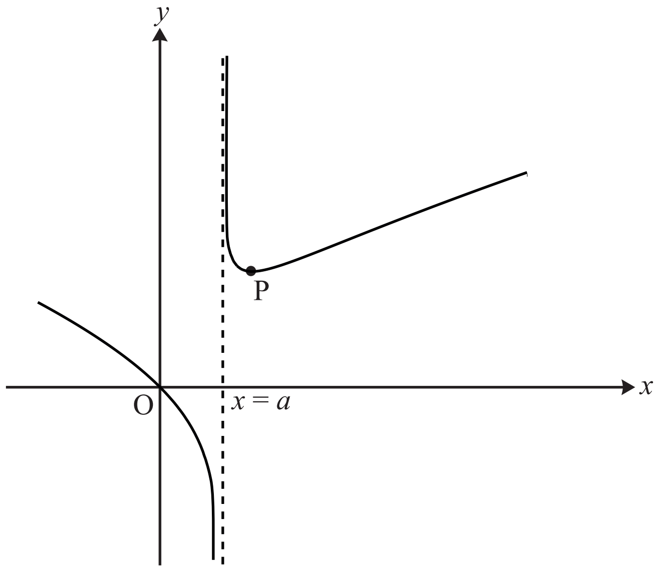

Fig. 9

(i) Write down the value of a. **[1]**

(ii) Show that $\dfrac{dy}{dx} = \dfrac{4x^3 - 3x^2}{3y^2(2x-1)^2}$.

Hence find the coordinates of the turning point P, giving the y-coordinate to 3 significant figures. **[9]**

(iii) Show that the substitution $u = 2x - 1$ transforms $\displaystyle\int \frac{x}{\sqrt[3]{2x-1}}\,dx$ to $\displaystyle\frac{1}{4}\int \left(u^{\frac{2}{3}} + u^{-\frac{1}{3}}\right) du$.

Hence find the exact area of the region enclosed by the curve $y^3 = \dfrac{x^3}{2x-1}$, the x-axis and the lines $x = 1$ and $x = 4.5$. **[8]**

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4753/01 Methods for Advanced Mathematics (C3)

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Candidate forename		Candidate surname	

Centre number						Candidate number				

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Section A (36 marks)

1 (i)	
1 (ii)	

2 (i)	
2 (ii)	

3 (i)	
3 (ii)	

3 (iii)

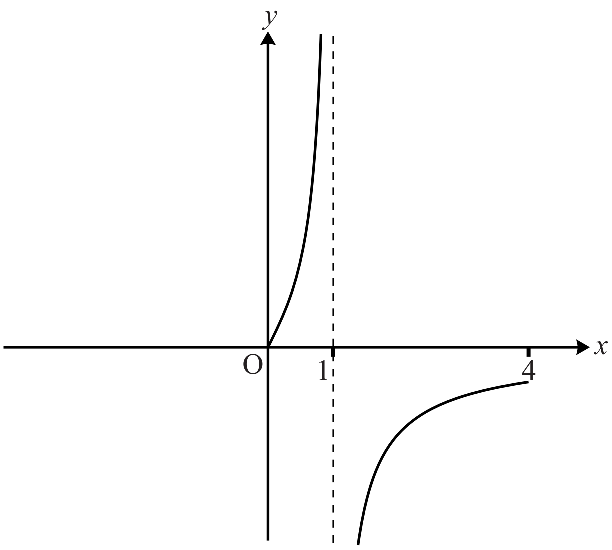
4

5

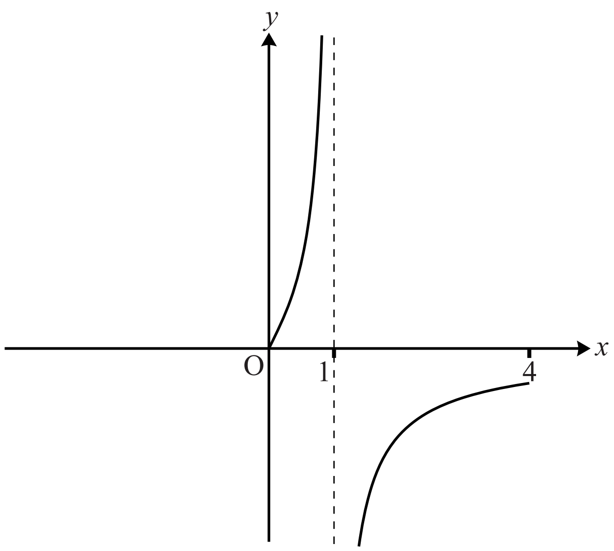
6	

7 (i)

7 (ii)



Spare copy of graph for question 7(ii)

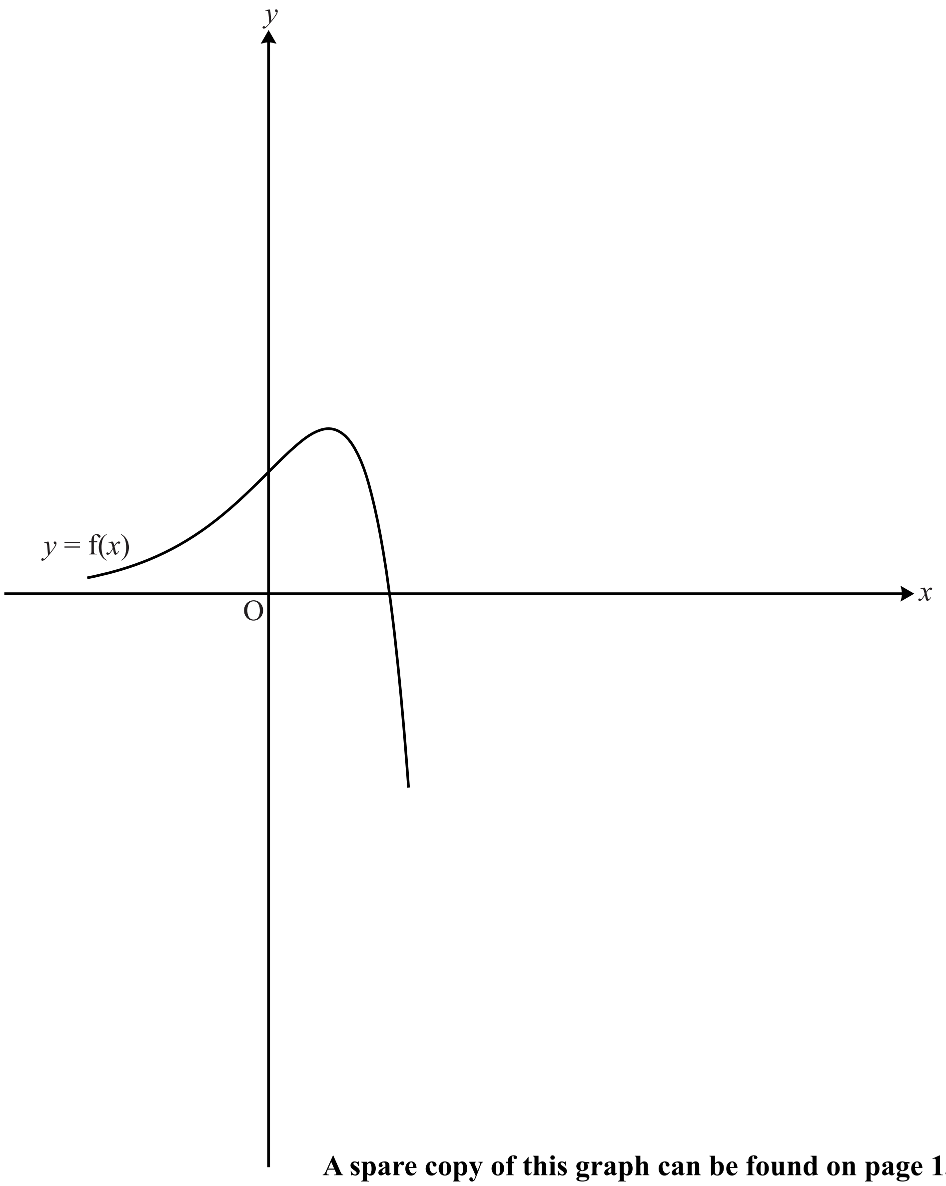


Section B (36 marks)

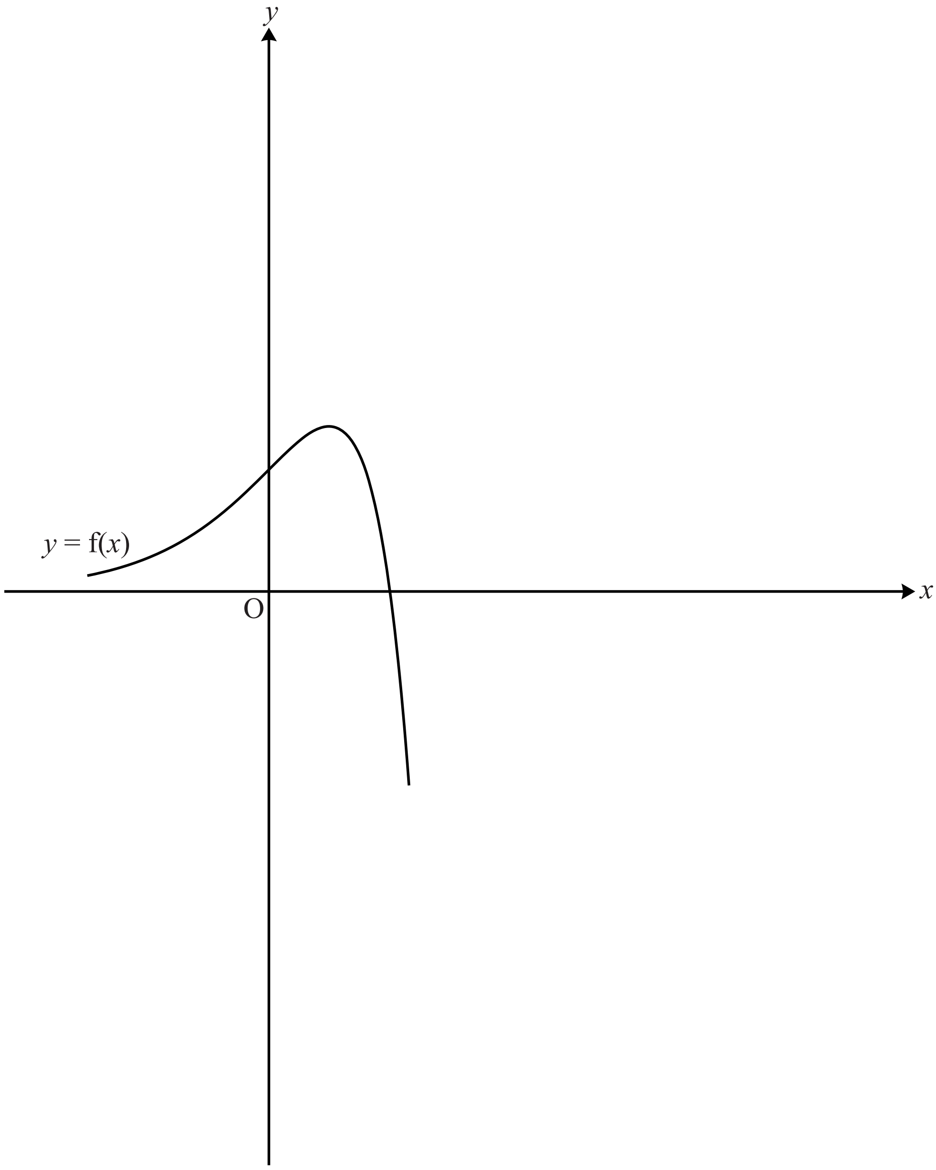
8 (i)	
8 (ii)	

8 (iii)

8 (iv)



A spare copy of this graph can be found on page 13.

8 (iv)	**Spare copy of graph for question 8(iv)** 
8 (v)	

9 (i)	
9 (ii)	
	(answer space continued on next page)

9 (ii)	**(continued)**
9 (iii)	
	(answer space continued on next page)

9 (iii)	**(continued)**

RECOGNISING ACHIEVEMENT

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GCE

Mathematics (MEI)

Advanced GCE

Unit **4753:** Methods for Advanced Mathematics

Mark Scheme for June 2013

Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	

Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

a. Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c. The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.
Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d. When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e. The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f. Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g. Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

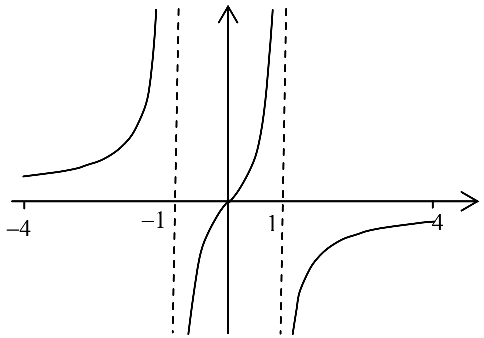
NB Follow these maths-specific instructions rather than those in the assessor handbook.

h. For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

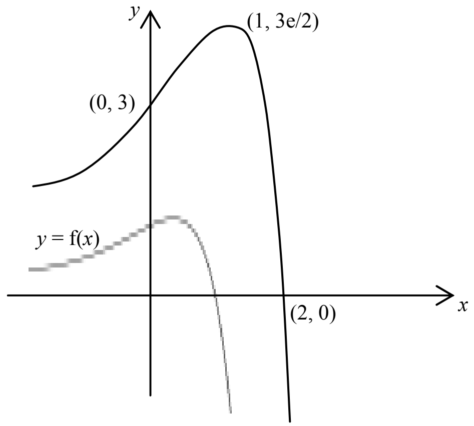
Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance
1	(i)		$a = ½$ $b = 1$	B1 B1 **[2]**	or 0.5
1	(ii)		$½\,\lvert x+1\rvert = \lvert x\rvert$ $\Rightarrow\ ½\,(x+1) = x,$ $\Rightarrow x = 1, y = 1$ or $½\,(x+1) = -x,$ $\Rightarrow x = -1/3, y = 1/3$	 M1 A1 M1 A1	 o.e. ft their a ($\neq 0$) , b (but allow recovery to correct values) or verified by subst $x = 1, y = 1$ into $y = ½\lvert x+1\rvert$ and $y = \lvert x\rvert$ unsupported answers M0A0 o.e., ft their a. b ; or verified by subst $(-1/3, 1/3)$ into $y = ½\,\lvert x+1\rvert$ and $y = \lvert x\rvert$ or 0.33, −0.33 or better unsupported answers M0A0
			or $¼\,(x+1)^2 = x^2$ $\Rightarrow 3x^2 - 2x - 1 = 0$ $\Rightarrow x = -1/3$ or 1 $y = 1/3$ or 1	 M1 M1ft A1 A1 **[4]**	 ft their a and b obtaining a quadratic = 0,ft their previous line, but must have an x^2 term SC3 for (1, 1) (−1/3, 1/3) and one or more additional points
2	(i)		$n^3 - n = n(n^2 - 1)$ $= n(n-1)(n+1)$	B1 B1 **[2]**	two correct factors
2	(ii)		$n-1$, n and $n+1$ are consecutive integers so at least one is even, and one is div by 3 [$\Rightarrow$ $n^3 - n$ is div by 6]	B1 B1 **[2]**	
3	(i)		Range is $-1 \le y \le 3$	M1 A1 **[2]**	−1, 3 $-1 \le y \le 3$ or $-1 \le f(x) \le 3$ or [−1, 3] (not −1 to 3, $-1 \le x \le 3$, $-1 < y < 3$ etc)

Question			Answer	Marks	Guidance
3	(ii)		$y = 1 - 2\sin x \;\; x \leftrightarrow y$		[can interchange x and y at any stage]
			$x = 1 - 2\sin y \Rightarrow x - 1 = -2\sin y$	M1	attempt to re-arrange
			$\Rightarrow \quad \sin y = (1 - x)/2$	A1	o.e. e.g. $\sin y = (x - 1)/(-2)$ (or $\sin x = (y - 1)/(-2)$)
			$\Rightarrow \quad y = \arcsin\,[(1 - x)/2]$	A1	or $f^{-1}(x) = \arcsin\,[(1 - x)/2]$, not x or $f^{-1}(y) = \arcsin[1 - y)/2]$ (viz must have swapped x and y for final 'A' mark).
				[3]	$\arcsin\,[(x - 1)/-2]$ is A0
3	(iii)		$f'(x) = -2\cos x$	M1	condone $2\cos x$
			$\Rightarrow \quad f'(0) = -2$	A1	cao
			$\Rightarrow \quad$ gradient of $y = f^{-1}(x)$ at $(1, 0) = -½$	A1	not $1/-2$
				[3]	
4			$V = \pi h^2 \Rightarrow \mathrm{d}V/\mathrm{d}h = 2\pi h \Rightarrow$	M1A1	if derivative $2\pi h$ seen without $\mathrm{d}V/\mathrm{d}h = \ldots$ allow M1A0
			$\mathrm{d}V/\mathrm{d}t = \mathrm{d}V/\mathrm{d}h \times \mathrm{d}h/\mathrm{d}t$	M1	soi ; o.e. – any correct statement of the chain rule using V, h and t – condone use of a letter other than t for time here
			$\mathrm{d}V/\mathrm{d}t = 10$	B1	soi; if a letter other than t used (and not defined) B0
			$\mathrm{d}h/\mathrm{d}t = 10/(2\pi \times 5) = 1/\pi$	A1	or 0.32 or better, mark final answer
				[5]	
5			$y = \ln\left(\sqrt{\dfrac{2x-1}{2x+1}}\right) = \dfrac{1}{2}(\ln(2x-1) - \ln(2x+1))$	M1	use of $\ln(a/b) = \ln a - \ln b$
				M1	use of $\ln\sqrt{c} = ½ \ln c$
			$\Rightarrow \quad \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{2}\left(\dfrac{2}{2x-1} - \dfrac{2}{2x+1}\right)$	A1	o.e.; correct expression (if this line of working is missing, M1M1A0A0)
			$= \dfrac{1}{2x-1} - \dfrac{1}{2x+1} \; *$	A1	**NB AG**
				[4]	for alternative methods, see additional solutions

Question			Answer	Marks	Guidance
6			$\int_0^{\pi/2} \frac{\sin 2x}{3+\cos 2x}\,\mathrm{d}x = \left[-\frac{1}{2}\ln(3+\cos 2x)\right]_0^{\pi/2}$	M1 A2	k ln(3 + cos 2x) – ½ ln(3 + cos 2x)
			or u = 3 + cos 2x, du = – 2sin2x dx	M1	o.e. e.g. du/dx = – 2sin2x or if v = cos2x, dv = – 2sin2xdx o.e. condone 2sin2x dx
			$\int_0^{\pi/2} \frac{\sin 2x}{3+\cos 2x}\,\mathrm{d}x = \int_4^2 -\frac{1}{2u}\,\mathrm{d}u$	A1	$\int -\frac{1}{2u}\mathrm{d}u$, or if v = cos2x, $\int -\frac{1}{2(3+v)}\mathrm{d}v$
			$= \left[-\frac{1}{2}\ln u\right]_4^2$	A1	[– ½ ln$u$] or [– ½ ln(3 + v)] ignore incorrect limits
			= – ½ ln 2 + ½ ln 4	A1	from correct working o.e. e.g. – ½ ln(3+cos(2.π/2)) + ½ ln(3 + cos(2.0))
			= ½ ln (4/2)		o.e. required step for final A1, must have evaluated to 4 and 2 at this stage
			= ½ ln 2 *	A1 **[5]**	**NB AG**
7	(i)		$\mathrm{f}(-x) = \frac{2(-x)}{1-(-x)^2}$	M1	substituting –x for x in f(x)
			$= -\frac{2x}{1-x^2} = -\mathrm{f}(x)$	A1 **[2]**	
7	(ii)			M1 A1 **[2]**	Recognisable attempt at a half turn rotation about O Good curve starting from x = – 4, asymptote x = – 1 shown on graph. (Need not state – 4 and – 1 explicitly as long as graph is reasonably to scale.) Condone if curve starts to the left of x = – 4.

Question			Answer	Marks	Guidance
8	(i)		(1, 0) and (0, 1)	B1B1 **[2]**	$x = 0, y = 1$; $y = 0, x = 1$
8	(ii)		$\mathrm{f}'(x) = 2(1-x)\mathrm{e}^{2x} - \mathrm{e}^{2x}$	B1	$\mathrm{d}/\mathrm{d}x(\mathrm{e}^{2x}) = 2\mathrm{e}^{2x}$
				M1	product rule consistent with their derivatives
			$= \mathrm{e}^{2x}(1-2x)$	A1	correct expression, so $(1-x)\mathrm{e}^{2x} - \mathrm{e}^{2x}$ is B0M1A0
			$\mathrm{f}'(x) = 0$ when $x = ½$	M1dep	setting their derivative to 0 dep 1st M1
				A1cao	$x = ½$
			$y = ½\,\mathrm{e}$	B1 **[6]**	allow $½\,\mathrm{e}^1$ isw
8	(iii)		$A = \int_0^1 (1-x)\mathrm{e}^{2x}\,\mathrm{d}x$	B1	correct integral and limits; condone no dx (limits may be seen later)
			$u = (1-x)$, $u' = -1$, $v' = \mathrm{e}^{2x}$, $v = ½\,\mathrm{e}^{2x}$	M1	u, u', v', v, all correct; or if split up $u = x$, $u' = 1$, $v' = \mathrm{e}^{2x}$, $v = ½\,\mathrm{e}^{2x}$
			$\Rightarrow \quad A = \left[\frac{1}{2}(1-x)\mathrm{e}^{2x}\right]_0^1 - \int_0^1 \frac{1}{2}\mathrm{e}^{2x}.(-1)\mathrm{d}x$	A1	condone incorrect limits; or, from above, ...$\left[\frac{1}{2}x\mathrm{e}^{2x}\right]_0^1 - \int_0^1 \frac{1}{2}\mathrm{e}^{2x}\mathrm{d}x$
			$= \left[\frac{1}{2}(1-x)\mathrm{e}^{2x} + \frac{1}{4}\mathrm{e}^{2x}\right]_0^1$	A1	o.e. if integral split up; condone incorrect limits
			$= ¼\,\mathrm{e}^2 - ½ - ¼$		
			$= ¼\,(\mathrm{e}^2 - 3)$ *	A1cao **[5]**	**NB AG**

Question			Answer	Marks	Guidance
8	(iv)		$g(x) = 3f(\frac{1}{2}x) = 3(1 - \frac{1}{2}x)e^x$	B1	o.e; mark final answer
				B1	through (2,0) and (0,3) – condone errors in writing coordinates (e.g. (0,2)).
				B1dep	reasonable shape, dep previous B1
				B1	TP at (1, 3e/2) or (1, 4.1) (or better). (Must be evidence that $x = 1$, $y = 4.1$ is indeed the TP − appearing in a table of values is not enough on its own.)
				[4]	
8	(v)		$6 \times \frac{1}{4}(e^2 - 3)$ [$= 3(e^2 - 3)/2$]	B1 **[1]**	o.e. mark final answer

Question			Answer	Marks	Guidance
9	(i)		$a = ½$	B1 **[1]**	allow $x = ½$
9	(ii)		$y^3 = \dfrac{x^3}{2x-1}$		
			$\Rightarrow \quad 3y^2\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{(2x-1)3x^2 - x^3.2}{(2x-1)^2}$	B1 M1 A1	$3y^2\mathrm{d}y/\mathrm{d}x$ Quotient (or product) rule consistent with their derivatives; $(v\,\mathrm{d}u + u\mathrm{d}v)/v^2$ M0 correct RHS expression – condone missing bracket
			$= \dfrac{6x^3 - 3x^2 - 2x^3}{(2x-1)^2} = \dfrac{4x^3 - 3x^2}{(2x-1)^2}$	A1	
			$\Rightarrow \quad \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{4x^3 - 3x^2}{3y^2(2x-1)^2}$ *	A1	**NB AG** penalise omission of bracket in QR at this stage
			$\mathrm{d}y/\mathrm{d}x = 0$ when $4x^3 - 3x^2 = 0$	M1	
			$\Rightarrow x^2(4x - 3) = 0$, $x = 0$ or ¾	A1	if in addition $2x - 1 = 0$ giving $x = ½$, A0
			$y^3 = (¾)^3/½ = 27/32$,	M1	must use $x = ¾$; if (0, 0) given as an additional TP, then A0
			$y = 0.945$ (3sf)	A1 **[9]**	can infer M1 from answer in range 0.94 to 0.95 inclusive

Question			Answer	Marks	Guidance
9	(iii)		$u = 2x - 1 \Rightarrow du = 2dx$		
			$\int \frac{x}{\sqrt[3]{2x-1}}dx = \int \frac{\frac{1}{2}(u+1)}{u^{1/3}}\frac{1}{2}du$	M1	$\frac{\frac{1}{2}(u+1)}{u^{1/3}}$ if missing brackets, withhold A1
				M1	× ½ du condone missing du here, but withhold A1
			$= \frac{1}{4}\int \frac{u+1}{u^{1/3}}du = \frac{1}{4}\int (u^{2/3} + u^{-1/3})du$ *	A1	**NB AG**
			area $= \int_1^{4.5} \frac{x}{\sqrt[3]{2x-1}}dx$	M1	correct integral and limits – may be inferred from a change of limits andP their attempt to integrate (their) ¼ ($u^{2/3} + u^{-1/3}$)
			when $x = 1$, $u = 1$, when $x = 4.5$, $u = 8$	A1	u = 1, 8 (or substituting back to x's and using 1 and 4.5)
			$= \frac{1}{4}\int_1^8 (u^{2/3} + u^{-1/3})du$		
			$= \frac{1}{4}\left[\frac{3}{5}u^{5/3} + \frac{3}{2}u^{2/3}\right]_1^8$	B1	$\left[\frac{3}{5}u^{5/3} + \frac{3}{2}u^{2/3}\right]$ o.e. e.g. [$u^{5/3}$/(5/3) + $u^{2/3}$/(2/3)]
			$= \frac{1}{4}\left[\frac{96}{5} + 6 - \frac{3}{5} - \frac{3}{2}\right]$	A1	o.e. correct expression (may be inferred from a correct final answer)
			$= 5\frac{31}{40} = 5.775$ or $\frac{231}{40}$	A1	cao, must be exact; mark final answer
				[8]	

Additional Solutions

Question			Answer	Marks	Guidance
5		(1)	$y=\ln\left(\sqrt{\frac{2x-1}{2x+1}}\right)=\frac{1}{2}\ln\left(\frac{2x-1}{2x+1}\right)$	M1	ln √c = ½ ln c used
			$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2}\frac{1}{\left(\frac{2x-1}{2x+1}\right)}\frac{(2x+1)2-(2x-1)2}{(2x+1)^2}$	A2	fully correct expression for the derivative
			$=\frac{1}{2}\frac{2x+1}{2x-1}\frac{4}{(2x+1)^2}=\frac{2}{(2x-1)(2x+1)}$		
			$\frac{1}{2x-1}-\frac{1}{2x+1}=\frac{2x+1-(2x-1)}{(2x-1)(2x+1)}$ $=\frac{2}{(2x-1)(2x+1)}$		
			$\frac{1}{2x-1}-\frac{1}{2x+1}=\frac{2x+1-(2x-1)}{(2x-1)(2x+1)}=\frac{2}{(2x-1)(2x+1)}$	A1 **[4]**	simplified and shown to be equivalent to $\frac{1}{2x-1}-\frac{1}{2x+1}$
5		(2)	$y=\ln\left(\sqrt{\frac{2x-1}{2x+1}}\right)=\ln\sqrt{2x-1}-\ln\sqrt{2x+1}$	M1	ln(a/b) = ln a – ln b used
			$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{\sqrt{2x-1}}\frac{1}{2}(2x-1)^{-1/2}.2-\frac{1}{\sqrt{2x+1}}\frac{1}{2}(2x+1)^{-1/2}.2$	A2	fully correct expression
			$=\frac{1}{2x-1}-\frac{1}{2x+1}$	A1 **[4]**	simplified and shown to be equivalent to $\frac{1}{2x-1}-\frac{1}{2x+1}$

Question			Answer	Marks	Guidance
5		**(3)**	$y=\ln\left(\sqrt{\dfrac{2x-1}{2x+1}}\right)$		
			$\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{1}{\sqrt{\dfrac{2x-1}{2x+1}}}\dfrac{1}{2}\left(\dfrac{2x-1}{2x+1}\right)^{-1/2}\dfrac{(2x+1)2-(2x-1)2}{(2x+1)^2}$ or	M1	$\dfrac{1}{u}\times$ their u' where $u=\sqrt{\dfrac{2x-1}{2x+1}}$ or $\dfrac{\sqrt{2x-1}}{\sqrt{2x+1}}$ (any attempt at u' will do)
			$\dfrac{1}{\dfrac{\sqrt{2x-1}}{\sqrt{2x+1}}}\dfrac{\sqrt{2x+1}.\frac{1}{2}.2(2x-1)^{-1/2}-\sqrt{2x-1}.\frac{1}{2}.2(2x+1)^{-1/2}}{\sqrt{2x+1}^2}$	A2	o.e. any completely correct expression for the derivative
			$=\dfrac{1}{2}\left(\dfrac{2x+1}{2x-1}\right)\dfrac{4}{(2x+1)^2}=\dfrac{2}{(2x-1)(2x+1)}$		or $=\dfrac{\sqrt{2x+1}}{\sqrt{2x-1}}\dfrac{(2x+1)-(2x-1)}{(2x+1)^{3/2}(2x-1)^{1/2}}=\dfrac{2}{(2x+1)(2x-1)}$
			$\dfrac{1}{2x-1}-\dfrac{1}{2x+1}=\dfrac{(2x+1)-(2x-1)}{(2x-1)(2x+1)}=\dfrac{2}{(2x-1)(2x+1)}$	A1 **[4]**	simplified and correctly shown to be equivalent to $\dfrac{1}{2x-1}-\dfrac{1}{2x+1}$
9	**(ii)**	**(1)**	$y=\dfrac{x}{(2x-1)^{1/3}}$		
			$\Rightarrow\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{(2x-1)^{1/3}.1-x.(1/3)(2x-1)^{-2/3}.2}{(2x-1)^{2/3}}$	M1 A1	quotient rule or product rule on y – allow one slip correct expression for the derivative
			$=\dfrac{6x-3-2x}{3(2x-1)^{4/3}}=\dfrac{4x-3}{3(2x-1)^{4/3}}$	M1 A1	factorising or multiplying top and bottom by $(2x-1)^{2/3}$
			$=\dfrac{(4x-3)x^2}{3y^2(2x-1)^{2/3}(2x-1)^{4/3}}=\dfrac{4x^3-3x^2}{3y^2(2x-1)^2}$	A1	establishing equivalence with given answer **NB AG**

Question			Answer	Marks	Guidance
9	(ii)	(2)	$y=\left(\dfrac{x^3}{(2x-1)}\right)^{1/3}$		
			$\Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{1}{3}\left(\dfrac{x^3}{(2x-1)}\right)^{-2/3}\dfrac{(2x-1).3x^2-x^3.2}{(2x-1)^2}$	B1	$\dfrac{1}{3}\left(\dfrac{x^3}{(2x-1)}\right)^{-2/3}\times\ldots$
				M1A1	$\ldots\times\dfrac{(2x-1).3x^2-x^3.2}{(2x-1)^2}$
			$=\dfrac{1}{3}\dfrac{4x^3-3x^2}{x^2(2x-1)^{4/3}}=\dfrac{4x-3}{3(2x-1)^{4/3}}$	A1	
			$=\dfrac{(4x-3)x^2}{3y^2(2x-1)^{2/3}(2x-1)^{4/3}}=\dfrac{4x^3-3x^2}{3y^2(2x-1)^2}$	A1	establishing equivalence with given answer **NB AG**
9	(ii)	(3)	$y^3(2x-1)=x^3$		
			$3y^2\dfrac{dy}{dx}(2x-1)+y^3.2=3x^2$	B1	$\mathrm{d}/\mathrm{d}x(y^3)=3y^2(\mathrm{d}y/\mathrm{d}x)$
				M1	product rule on $y^3(2x-1)$ or $2xy^3$
				A1	correct equation
			$\dfrac{dy}{dx}=\dfrac{3x^2-2y^3}{3y^2(2x-1)}$		
			$=\dfrac{3x^2-2\dfrac{x^3}{(2x-1)}}{3y^2(2x-1)}$	M1	subbing for $2y^3$
			$=\dfrac{3x^2(2x-1)-2x^3}{3y^2(2x-1)^2}=\dfrac{6x^3-3x^2-2x^3}{3y^2(2x-1)^2}=\dfrac{4x^3-3x^2}{3y^2(2x-1)^2}$	A1	**NB AG**

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GCE

Mathematics (MEI)

Advanced GCE **A2 7895-8**

Advanced Subsidiary GCE **AS 3895-8**

OCR Report to Centres

June 2013

4753 Methods for Advanced Mathematics (C3 Written Examination)

General Comments

This question paper proved to be accessible, and many candidates scored over 65 marks. There were few candidates who scored below 25 marks. Virtually all candidates had enough time to complete the paper, though some questions, especially question 5, rewarded candidates who selected efficient methods, and there were a lot of candidates who required additional sheets to make further attempts at questions. It should be pointed out to candidates that examiners mark the last of a number of equally complete attempts (unless instructed otherwise), and this is not always the solution which scores the most marks.

There is a lot of calculus in this question paper, and the standard methods for differentiating and integrating were usually applied well. Notation is, however, important and candidates who miss out d*x*'s or d*u*'s (especially when integrating by substitution), or essential brackets, can lose marks. This aspect has improved over the years but there are still candidates who do not understand the need for accurate notation and lose marks as a consequence. Other questions with given answers, such as question 5, require enough working to be shown as evidence that a correct method is being used, and it is particularly important to emphasise this to more able candidates, who are capable of processing steps in their heads which are nevertheless required to be written down for complete solutions. Usually, candidates who try to 'fiddle' solutions lose marks – question 6 is a good example of this.

The presentation of solutions varies enormously from candidates who write fluent, logical mathematics to those who offer disjointed, unconnected statements which lack any logical coherence, and leave others to decide whether they constitute a correct solution. While this is often not penalised – see the example offered below for question 7(i) – it would be nice to see more evidence of mathematics perceived as a true language, with statements linked with appropriate logical connectives such as 'equals' and 'which implies'.

Comments on Individual Questions

1(i) Some candidates were able to write down the correct values of a and b. Those who chose to use transformation arguments sometimes confused the stretch (1/2 or 2) and the translation (+1 or −1). Others chose to substitute the coordinates of specific points, with variable success.

1(ii) Most candidates, who knew what they were doing here either used $\frac{1}{2}(x + 1) = \pm x$ or squared both sides to find a quadratic in x. In the latter approach, some forgot to square the ½ and got the wrong quadratic. Examiners followed through their values for a and b. Some candidates omitted the y-coordinates. Candidates who found (1, 1) without showing a valid method got no marks, and there was evidence of the usual mistakes in using modulus, such as $|x + 1|= |x| + 1$, etc.

2(i) Many candidates failed to factorise the $n^2 – 1$, leaving their answer as $n(n^2 – 1)$. This rendered the second part of the question very difficult.

2(ii) There were two ideas needed here, the realisation that $n – 1$, n and $n + 1$ were consecutive integers, and that the product contained factors 2 and 3. Many candidates argued that the product had to be even, but this was not enough to gain credit. Others, predictably, verified the result with a few values of n, often describing this as 'proof by exhaustion'.

3(i) This was generally well done, with one mark awarded for −1 and 3 seen, and one for the correct notation. Some used x instead of y or f(x), and others confused domain and range.

3(ii) Most candidates are well practiced at finding inverses, and were familiar with arcsine, gaining full marks here. Leaving the result as $y = \arcsin((x – 1)/−2)$ lost the final A1. Very occasionally, candidates gave the answer as 1/f(x) or f'(x).

3(iii) Nearly all candidates found f'(x) and f'(0) correctly. The gradient of the inverse function was less successful. Many confused this with the condition for perpendicularity and gave the answer ½ instead of - 1/2 . Those who tried to differentiate $f^{-1}(x)$ directly had little success.

4(i) This proved to be an accessible 5 marks, with many candidates getting the question fully correct. Of those who did not, dh/dt = 10 (instead of dV/dt) was quite a common misconception; some tried to find dh/dV but failed to handle the constant of $1/\sqrt{\pi}$ correctly; and a surprising number finished off by saying that $10/10\pi = \pi$ instead of $1/\pi$.

5 Some candidates spotted the trick of simplifying the given function to get $y = ½ \ln(2x – 1) – ½ \ln(2x + 1)$ before differentiating, and thereby made lives considerably easier for themselves! However, writing the answer down from here omitted the vital 2 x ½ working and lost two marks. Those who started differentiating from $y = \ln(\sqrt{(2x – 1)} – \ln(\sqrt{(2x + 1)})$ needed to convince that they were using a chain rule on $\sqrt{u}$, where $u = 2x – 1$. Some tenacious candidates even managed to differentiate the given function correctly without these preliminaries, but made life hard for themselves.

6 The error d/dx (cos 2x) = 2sin 2x proved costly here, earning only a consolation M1; many also wrote the limits the wrong way round on the integral, and scored 3 out of 5, unless they 'lost' the negative sign, and scored M1 only. Many candidates seem unaware that swapping limits dealt with the negative sign. We also needed to see some evidence of why ln 4 – ln 2 = 2 to score the final A1.

7(i) This was generally well answered, though the flow of the argument was not always apparent. Many candidates write down arguments such as:

$f(−x) = −f(x)$
$2(−x)/(1 – (−x)^2) = −2x/(1 – x^2)$,

rather than the more convincing:

$f(−x) = 2(−x)/(1 – (−x)^2)$
$\quad = −2x/(1 – x^2) = −f(x)$

Examiners condone this sort of logical error where possible, but candidates should be encouraged to frame such arguments correctly, with the use of implication signs if possible. Using 'RTP' for 'required to prove' might help to prevent candidates from arguing from the result they are trying to establish.

Those candidates who failed to scored 2 marks here either made errors in writing f(−x) as $−2x/(1 − −(x)^2)$, wrote that for odd functions $f(−x) \neq f(x)$, or verified using one value of x.

7(ii) Many candidates scored full marks here. The asymptote need to be indicated for the A1, and occasionally the section of curve from x = 0 to x = −1 was omitted.

8(i) The points of intersection were a write-down for many candidates. Weaker attempts failed to solve $(1 – x) e^{2x} = 0$ convincingly.

8(ii) This proved to be an accessible 6 marks for candidates. The derivative of e^{2x} and the product rule were generally correct, and deriving $x = ½$ and $y = e^{1/2}$ was straightforward, though many did not simplify the derivative to $e^{2x} – 2xe^{2x}$ immediately. Some candidates approximated for $e^{1/2}$ and lost a mark.

8(iii) Most candidates applied integration by parts to either $\int(1 – x)\, e^{2x}dx$ or $\int x\, e^{2x}dx$, using appropriate u, v', u' and v. Sign and/or bracket errors sometimes meant they failed to derive the correct result, but many were fully correct.

8(iv) This part proved to be quite demanding. Deriving the formula for g(x) was rarely correctly done. Common errors were an extra factor of 3 and an incorrect exponent. Most graphs showed the correct points of intersection (0, 3) and (2, 0), but the turning point was quite often incorrect or missing, and the shape failed to convince.

8(v) Those, of the relatively few candidates, who got this correct just wrote down $2 \times 3 \times ¼\, (e^2 – 3)$. Some tried to integrate g(x), with little success.

9(i) Nearly all candidates gained this mark for the asymptote.

9(ii) Candidates tended to score heavily on this part. The implicit differentiation of y^3 was usually correct (albeit introduced into solutions belatedly), and the quotient rule was done well, though occasionally omission of brackets was penalised. Those who cube rooted and differentiated often succeeded in arriving at the given derivative. Another approach was to multiplying across before differentiating implicitly, but with required candidates to substitute for y to deduce the required form for the derivative. Finding x = ¾ for the turning point from the given derivative was straightforward, but some failed to find the correct y-coordinate by omitting the necessary cube root.

9(iii) There were plenty of accessible marks here as well. The first three marks, for transforming the integral to the variable u, were usually negotiated successfully, although poor notation – omitting du's or brackets – was sometimes penalised in the A1 mark. The second half involved evaluating the given integral with the correct limits. Some calculated the correct limits, but made errors in the integral (or forgot to integrate altogether). However, a reasonable number of candidates managed to do this work without errors. A rather curious misconception was to cube the correct value of the integral, because the function was presented implicitly in terms of y^3.

Unit level raw mark and UMS grade boundaries June 2013 series
AS GCE / Advanced GCE / AS GCE Double Award / Advanced GCE Double Award

GCE Mathematics (MEI)

		Max Mark	a	b	c	d	e	u
4751/01 (C1) MEI Introduction to Advanced Mathematics	Raw	72	62	56	51	46	41	0
	UMS	100	80	70	60	50	40	0
4752/01 (C2) MEI Concepts for Advanced Mathematics	Raw	72	54	48	43	38	33	0
	UMS	100	80	70	60	50	40	0
4753/01 (C3) MEI Methods for Advanced Mathematics with Coursework: Written Paper	Raw	72	58	52	46	40	33	0
4753/02 (C3) MEI Methods for Advanced Mathematics with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4753/82 (C3) MEI Methods for Advanced Mathematics with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
4753 (C3) MEI Methods for Advanced Mathematics with Coursework	UMS	100	80	70	60	50	40	0
4754/01 (C4) MEI Applications of Advanced Mathematics	Raw	90	66	59	53	47	41	0
	UMS	100	80	70	60	50	40	0
4755/01 (FP1) MEI Further Concepts for Advanced Mathematics	Raw	72	63	57	51	45	40	0
	UMS	100	80	70	60	50	40	0
4756/01 (FP2) MEI Further Methods for Advanced Mathematics	Raw	72	61	54	48	42	36	0
	UMS	100	80	70	60	50	40	0
4757/01 (FP3) MEI Further Applications of Advanced Mathematics	Raw	72	60	52	44	36	28	0
	UMS	100	80	70	60	50	40	0
4758/01 (DE) MEI Differential Equations with Coursework: Written Paper	Raw	72	62	56	51	46	40	0
4758/02 (DE) MEI Differential Equations with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4758/82 (DE) MEI Differential Equations with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
4758 (DE) MEI Differential Equations with Coursework	UMS	100	80	70	60	50	40	0
4761/01 (M1) MEI Mechanics 1	Raw	72	57	49	41	33	25	0
	UMS	100	80	70	60	50	40	0
4762/01 (M2) MEI Mechanics 2	Raw	72	50	43	36	29	22	0
	UMS	100	80	70	60	50	40	0
4763/01 (M3) MEI Mechanics 3	Raw	72	64	56	48	41	34	0
	UMS	100	80	70	60	50	40	0
4764/01 (M4) MEI Mechanics 4	Raw	72	56	49	42	35	29	0
	UMS	100	80	70	60	50	40	0
4766/01 (S1) MEI Statistics 1	Raw	72	55	48	41	35	29	0
	UMS	100	80	70	60	50	40	0
4767/01 (S2) MEI Statistics 2	Raw	72	58	52	46	41	36	0
	UMS	100	80	70	60	50	40	0
4768/01 (S3) MEI Statistics 3	Raw	72	61	55	49	44	39	0
	UMS	100	80	70	60	50	40	0
4769/01 (S4) MEI Statistics 4	Raw	72	56	49	42	35	28	0
	UMS	100	80	70	60	50	40	0
4771/01 (D1) MEI Decision Mathematics 1	Raw	72	58	52	46	40	35	0
	UMS	100	80	70	60	50	40	0
4772/01 (D2) MEI Decision Mathematics 2	Raw	72	58	52	46	41	36	0
	UMS	100	80	70	60	50	40	0
4773/01 (DC) MEI Decision Mathematics Computation	Raw	72	46	40	34	29	24	0
	UMS	100	80	70	60	50	40	0
4776/01 (NM) MEI Numerical Methods with Coursework: Written Paper	Raw	72	56	50	44	38	31	0
4776/02 (NM) MEI Numerical Methods with Coursework: Coursework	Raw	18	14	12	10	8	7	0
4776/82 (NM) MEI Numerical Methods with Coursework: Carried Forward Coursework Mark	Raw	18	14	12	10	8	7	0
4776 (NM) MEI Numerical Methods with Coursework	UMS	100	80	70	60	50	40	0
4777/01 (NC) MEI Numerical Computation	Raw	72	55	47	39	32	25	0
	UMS	100	80	70	60	50	40	0
4798/01 (FPT) Further Pure Mathematics with Technology	Raw	72	57	49	41	33	26	0
	UMS	100	80	70	60	50	40	0

GCE Statistics (MEI)

		Max Mark	a	b	c	d	e	u
G241/01 (Z1) Statistics 1	Raw	72	55	48	41	35	29	0
	UMS	100	80	70	60	50	40	0
G242/01 (Z2) Statistics 2	Raw	72	55	48	41	34	27	0
	UMS	100	80	70	60	50	40	0
G243/01 (Z3) Statistics 3	Raw	72	56	48	41	34	27	0
	UMS	100	80	70	60	50	40	0